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# Continuation of Real Analytic Solutions of Partial Differential Equations up to Convex Conical Singularities(Algebraic Analysis and Number Theory)

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# Continuation of Real Analytic Solutions of Partial Differential Equations up to Convex Conical Singularities

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In his talk at the RIMS Seminar in December 1985, Kaneko gave the following conjecture (cf. [Kn3]):

**Kaneko's Conjecture.** Let  $P = D_t^2 - \Delta$  be the wave operator on the Euclidean  $n$  space  $\mathbf{R}^n$ . Let  $\Gamma$  be a closed convex proper cone of  $\mathbf{R}^n$  with vertex at the origin, sharp enough in a certain direction; i.e.,  $\Gamma$  is contained in  $\{x_1 \geq C|x_2|\}$  for a Euclidean coordinate  $(x_1, \dots, x_n)$  of  $\mathbf{R}^n$ , for a large  $C > 0$ . Let  $R > 0$  and set  $K = \{(x, t) \in \mathbf{R}^n \times \mathbf{R} \mid x \in \Gamma, |t| \leq R|x|\}$ . Then any real analytic solution to the wave equation  $Pu = 0$  defined outside  $K$  can be analytically continued up to the origin  $(0, 0)$  of  $\mathbf{R}^n \times \mathbf{R}$ .

We give an answer to this conjecture in a general context.

**Definition.** Let  $K$  be a closed subset of a real analytic manifold  $M$  of dimension  $n$ .  $K$  is said to be  $C^\alpha$ -convex at  $x \in M$  ( $1 \leq \alpha \leq \omega$ ) if there exist a neighborhood  $U$  of  $x$  and an open  $C^\alpha$ -immersion  $\phi : U \rightarrow \mathbf{R}^n$  such that  $\phi(U \cap K)$  is convex in  $\mathbf{R}^n$ .  $K$  is said to have a conical singularity at  $x$  if  $x \in K$  and the tangent cone  $C_x(K)$  is a closed proper cone of  $T_x M$ .

**Theorem 0.1.** Let  $K$  be a  $C^1$ -convex closed subset of a real analytic manifold  $M$ , having a conical singularity at  $x$ . Let  $P = P(x, D)$  be a second order differential operator with analytic coefficients defined in a neighborhood of  $x$ . Assume that  $P$  is of real principal type and is not elliptic. Then any real analytic solution to the equation  $Pu = 0$  defined outside  $K$  is analytically continued up to  $x$ .

In order to state a similar result for overdetermined systems of differential equations, we first recall the notion of a virtual bicharacteristic manifold of a system  $\mathcal{M}$  of differential equations.

Let  $V = \text{Char}(\mathcal{M})$ ;  $V^c$  denotes the complex conjugate of  $V$  with respect to  $T_M^* X$ . Let  $p \in V \cap (T_M^* X \setminus M)$ . Assume the following:

- (b.1)  $V$  is nonsingular at  $p$ .
- (b.2)  $V$  and  $V^c$  intersect cleanly at  $p$ ; i.e.,  $V \cap V^c$  is a smooth manifold and

$$T_p V \cap T_p V^c = T_p (V \cap V^c).$$

- (b.3)  $V \cap V^c$  is regular; i.e.,  $\omega|_{V \cap V^c} \neq 0$ , with  $\omega$  being the fundamental 1-form on  $T^* X$ .
- (b.4) The generalized Levi form of  $V$  has constant rank in a neighborhood of  $p$ .

Then one can define the virtual bicharacteristic manifold  $\Lambda_p$  of  $\mathcal{M}$  passing through  $p$  (cf. [SKK, Ch.III, Sect.2.4]). we assume

$$(b.5) \quad d\pi(T_p\Lambda_p) \neq \{0\}.$$

**Theorem 0.2.** *Let  $(K, x)$  be as in Theorem 0.1. Let  $\mathcal{M}$  be a system of differential equations defined in a neighborhood of  $x$ . Assume that  $\text{Char}(\mathcal{M}) \cap \pi^{-1}(x)$  has codimension  $\geq 2$  in  $\pi^{-1}(x)$  and that  $V = \text{Char}(\mathcal{M})$  satisfies conditions (b.1)—(b.5) at each point  $p$  of  $V \cap (T_M^*X \setminus M) \cap \pi^{-1}(x)$ . Then any real analytic solution to  $\mathcal{M}$  defined outside  $K$  is analytically continued up to  $x$ .*

**Corollary.** *Let  $(K, x)$  be as in Theorem 0.2. Let  $\mathcal{M}$  be an elliptic system of differential equations and assume that  $\text{Char}(\mathcal{M}) \cap \pi^{-1}(x)$  has codimension  $\geq 2$  in  $\pi^{-1}(x)$ . Then any solution  $u$  of  $\mathcal{M}$  defined outside  $K$  can be analytically continued up to  $x$ .*

*Remark.* Cf. [Kw], theorems 4 and 5, for general results on analytic continuation of the solutions of overdetermined systems of differential equations.

The following theorem is a generalization of Theorem 0.1 to higher order differential equations for  $K = \{x_0\}$ . Cf. Theorem 17 and Corollary 22 of [Kn2].

**Theorem 0.3.** *Let  $P = P(x, D)$  be a differential operator of real principal type. Assume that the polynomial  $f(x_0; \zeta)$  in  $\zeta$  has no elliptic factors. Then any real analytic solution to the equation  $Pu = 0$  defined in a neighborhood of  $x_0$  except  $x_0$  can be analytically continued on the whole of a neighborhood of  $x_0$ .*

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